Weak measure expansive homoclinic classes

Jumi Oh [(1)], Bowon Kang [(2)]

[(1)]Department of Mathematics, Sungkyunkwan University, Suwon, 16419, Korea.
+82-10-9417-2661 and ohjumi@skku.edu

[(2)] Department of Mathematics, Chungnam National University, Daejeon, 34134, Korea
+82-10-6382-3830 and

1. Introduction –A well known result by Ma\~n\'e says that a C^1 stably expansive diffeomorphism of a closed manifold is quasi-Anosov [1]. In [2] it was studied C^1 stably expansive properties of homoclinic classes. As a result it was proved that all of them are hyperbolic for C^1 generic diffeomorphism. Afterwards, it comes [3] where the C^1 stably hypothesis in [2] was removed. Namely every expansive homoclinic class of a C^1 generic diffeomorphism of a closed manifold is hyperbolic. The recent notion of measure expansive systems suggested to replace expansive by measure expansive in [3] in order to obtain similar result. This was done in [4] where every measure expansive homoclinic class of a C^1 generic diffeomorphism of a closed manifold was proved to be hyperbolic. On the other hand, [5,6] defined weak measure expansive systems. Then, it is natural to replace expansiveness in [3] by weak measure expansiveness in order to get similar conclusions.

2. Results and Discussion –
In dynamical systems, it is one of the main goal to study the stability of an orbit structure for given systems. The notion of expansiveness for a homeomorphism on a compact metric space X introduced by Utz [7] is important in the qualitative study of dynamical systems. A system is expansive if two orbits cannot remain close to each other under the action of the system. So it is natural to consider the stability of the system which has the expansiveness from the stable and generic point of view. We consider the weak measure expansive property on the homoclinic classes.

Theorem A. Every weak measure expansive homoclinic class of a C^1 generic diffeomorphism of a closed manifold is hyperbolic.

Next we present an stable version of this result. The basic example of a C^1 stable weak measure expansive set if the Smale Horseshoe. Notice that such an example is also a hyperbolic homoclinic class. Thus motivates our second result.

Theorem B. Every C^1 stably weak measure expansive homoclinic class of every diffeomorphism of a closed manifold is hyperbolic.

3. Conclusions - In this talk, we prove that every weak measure expansive homoclinic class of a C^1 generic diffeomorphism of a closed manifold is hyperbolic. Moreover, we also prove that every C^1 stably weak measure expansive homoclinic class of every diffeomorphism of a closed manifold is hyperbolic.

4. References
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