Iterated Petrov - Galerkin method with Regular Pairs for solving Fredholm Integral Equations of the Second Kind

S. Seminara [(1)], M. I. Troparevsky

Depto. de Matemática - Facultad de Ingeniería - Universidad de Buenos Aires
[(1)] +5491131471920 - seminarasilvia@gmail.com

1. Introduction – Classical Fredholm equations of the second kind are integral equations that appear in different areas of applied mathematics, sometimes as equivalent formulation of boundary value problems for ordinary and partial differential equations. Under proper conditions, they can be expressed in the form of an operator equation in a Banach space B, $u - Ku = f$, for $K : B \to B$ linear and compact and $f \in B$ a given function. Numerical solutions of these equations have been often developed in the context of Galerkin type schemes. Particularly, Petrov-Galerkin method has been proposed to solve this type of equations by projecting on appropriate sequences of finite dimensional subspaces. Through the notion of "regular pair" of subspaces, solvability and numerical stability of the approximation scheme is guaranteed. In the case of Hilbert spaces, the characterization of a regular pair is very simple because it can be related to the positive definitiveness of a correlation matrix, as it is shown in [1]. An interesting phenomenon called "superconvergence" was observed in the '70s by Sloan: once the approximations $u_n \in X_n$ are obtained, the convergence can be notably improved by means of an iteration of the method, $u*_n = f + Ku_n$, as it is shown in [2].

2. Numerical examples – In this work we first choose pairs of simple subspaces $\{X_n, Y_n\}$ generated by Legendre polynomials, that are regular for all $n \in N$, and show the goodness of the of approximations in some numerical examples with known solutions. Then, following [2], we improved the convergence by means of an iteration of the method and obtained a better approximation, even for small values of $n \in N$, and with very simple computation. Image 1 shows how the approximations improve as *n* is increased and, in Image 2, it can be seen that they are still better after the iteration procedure is performed.

4. Conclusions – A well-known method is applied, with quite simple calculations by choosing appropriate subspaces of projection. Iteration shows to be a very simple way for improving convergence in a remarkable way and new orders of convergence can be shown.

5. References

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[2] I. H. Sloan, "The iterated Galerkin method for integral equations of the second kind", Proceedings of the Centre for Mathematics and its Applications, 5, (1984), pp. 153-161.