Hyperbolic-Parabolic Balance Laws: Asymptotic Behavior and Gas Flows in Thermal Non-equilibrium

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1. Introduction – We consider a general class of partial differential equations from continuum mechanics. They are systems describing the balance of physical quantities such as mass, momentum, energy, etc. We are interested in systems with dissipation from viscosity, heat conduction, diffusion, damping, relaxation, and chemical reaction. Examples are Navier-Stokes equations for compressible fluids, Euler equations with damping, equations for gas flows in vibrational non-equilibrium, and equations for gas flows in both translational and vibrational non-equilibrium. A common feature of these examples is that they all have the continuity equation, which is non-dissipative. We extend those examples to a general framework: a general system with viscosity matrices and a lower order term, while the viscosity matrices and the Jacobian matrix of the lower order term are allowed to be rank deficient. We call systems in such form hyperbolic-parabolic balance laws.

We are interested in Cauchy problem of hyperbolic-parabolic balance laws, where the Cauchy data are small perturbations of a constant equilibrium state. In a recent publication [1], the author proposed a set of structural conditions that imply the existence of solution global in time. Under the same set of conditions, we can further obtain the L^2 decay rate of solution to the constant equilibrium state [2]. Under physical assumptions on the thermodynamic variables, the set of structural conditions are satisfied by examples such as Navier-Stokes equations, Euler equations with damping, and equations for gas flows in both translational and vibrational non-equilibrium. The goal of this work is to further study the asymptotic behaviour of solution under the structural conditions.

2. Results and Discussion – We have shown that there is an intrinsic difference between multi space dimensions and one space dimension (plane wave solutions) due to the fact that diffusion waves decay faster in multi space dimensions. In particular, it can be proved that an asymptotic solution in multi space dimension is the solution of the corresponding linear system, linearized around the constant equilibrium state, and with the same Cauchy data. The conclusion, however, is not true for one space dimension. In the latter nonlinear waves need to be included in the construction of asymptotic solution. Our result, in particular, applies to gas flows in translational and vibrational non-equilibrium.

3. Conclusions – We prove that in multi space dimensions, the Cauchy problem around a constant equilibrium state of a system of hyperbolic-parabolic balance laws is time-asymptotically a linear problem. The conclusion does not apply to one space dimension. Application to gas flows in translational and vibrational non-equilibrium is discussed.

4. References

[1] Y. Zeng, *J. Hyperbolic Differ. Equ*. **14**(2), (2017) pp 359-391.
[2] Y. Zeng, *Discrete Contin. Dyn. Syst.* **38**(1), (2018) pp 363-396.