C^2-robustly weak measure expansive diffeomorphisms

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1. Introduction – Under mainly C^1-topology, we have studied hyperbolicity using various properties for diffeomorphisms or flows. To study dynamics that are more similar (closer) to given dynamics, Many mathematicians are studying C^2-topology. Indeed, Pujal and Sambarino showed the following in [3]; Let f be a C^2 diffeomorphism and $\Lambda \subset \Omega(f)$ be a compact f-invariant set admitting a dominated splitting. Assume all periodic points in $\wedge$ are hyperbolic saddles. Then $\wedge = \wedge_1 \cup \wedge_2$, where $\wedge_1$ is a hyperbolic set and $\wedge_2$ is the union of finitely many pairwise disjoint normally hyperbolic circles $C_1, \ldots, C_k$ such that $f^{m_i}(C_i) = C_i$ and $f^{m_i} : C_i \to C_i$ is an irrational rotation for some $m_i \geq 1$ (m_i denotes the minimal number satisfying $f^{m_i}(C_i) = C_i$). From this, Sakai and Lee got the extended results in terms of shadowing property in [4] and [2], respectively. Also Artigue obtained the enhanced result as the view point of expansivity in [1]. In this talk, we want to generalize the Artigue's result.

2. Results and Discussion – Let M be a compact smooth n(≥ 2)-dimensional manifold without boundary. Put $\mathcal{M}$(M) as the set of all probability measures and $\mathcal{M}^*$(M) as the set of all nonatomic elements of $\mathcal{M}$(M). Waltes defined expansivity for the first time in the 1950s. Next Kato defined CW- expansivity in 1993, and Morlaes recently introduced other forms of expansivity which is called N-expansivity and measure expansivity. We have newly defined the concept of "*weak measure expansivity*", which is a generalization of Morales' definition in particular. Now, let us introduce the definition of weak measure expansivity; for any $\mu \in \mathcal{M}^*(M)$, f is called weak μ- expansive, if there is a finite partition $\mathcal{P} = \{A_1, \cdots,$
$A_k\}$ of M such that $\mu(\Gamma_{\mathcal{P}}(x)) = 0$ for all x ∈ M, where $\Gamma_{\mathcal{P}}(x) = \{y \in M : f^n(y) \in P(f^n(x)) \text{ for all } n \in \mathbb{Z}\}$ (here, P(x) means the element of $\mathcal{P}$ containing x ∈ M). Furthemore, we say that f is weak measure expansive if f is weak μ- expansivefor all $\mu \in \mathcal{M}^*(M)$. We will show the following theorems.

Theorem 1. If f is C^2-robustly weak measure expansive diffeomorphism on M and p is a periodic point with period π(p), then $D_p f^{\pi(p)}$ has eigenvalues λ_1, λ_2 with $|\lambda_1| < 1, |\lambda_2| > 1$.

Theorem 2. Every C^2-robustly weak measure expansive diffeomorphism on a compact surface is C^2-star.

3. Conclusions - The following theorem is what we want to prove as a conclusion in this talk using the above two theorems. Let $f: M \to M$ be a C^2-star diffeomorphism with Axiom A and no cycle condition. If f is C^2-robustly weak measure expansive diffeomorphism, then it is Q^2-Anosov.

4. References

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