A type of robust perturbation of continuum-wise expansiveness for vector fields

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1. Introduction – From Utz [1] expansiveness have been studied to hyperbolic systems. In fact, a diffeomorphism f of a compact smooth manifold M, Mane [2] proved that if a diffeomorphism f belongs to the C^1-interior of the set of expansive diffeomorphisms then f satisfies Axiom A and satisfies the quasi-transversality condition. Later, Sakai [3] proved that if a diffeomorphism f belongs to the C^1-interior of the set of continuum-wise expansive diffeomorphisms then f satisfies Axiom A and satisfies the quasi-transversality condition. Das, Lee and Lee [4] proved that if the chain recurrence set R(f) is robustly continuum-wise expansive then it is Axiom A without cycles. Morover, they showed that if the homoclinic class H(p) is continuum-wise expansive and it satisfies the chain condition then it is hyperbolic. According to the results, we consider vector fields which are an extended results of diffeomorphisms.

2. Results and Discussion - Let M be a closed smooth Riemannian manifold with $\dim M \geq 3$, and let d be the distance on M induced from a Riemannian metric $|\cdot|$ on the tangent bundle TM and denote by $\Xi(M)$ the set of C^1-vector fields on M endowed with the C^1-topology. Then every $X \in \Xi(M)$ generates a C^1-flow $X^t: M \times R \to M$ that is a C^1-map such that $X^t: M \to M$ is a diffeomorphism satisfying (i) $X^0(x) = x$, (ii) $X^t(X^s(x)) = X^{t+s}(x)$ for all $t, s \in R$, and $x \in M$. A point $\sigma \in M$ is singular such that $X^t(\sigma) = \sigma$ for all $t \in R$. We denote by Sing(X) the set of all singular point of X.
An increasing homeomorphism h:R→R with h(0)=0 which is called reparametrization. Denote Hom(R) by the set of all homeomorphisms of R. Let Rep(R)={h∈Hom(R): h is a reparametrization }. If A is a subset of M, C^0(A, R) denotes the set of real continuous maps defined on A.
Define H(A)={h:A→Rep(R): there is x_h∈A with $h(x_h) = id$ and $h(\cdot)(t) \in C^0(A, R)$, for all $t \in R$}, and if $t \in R$ and h∈H(A), $\Upsilon_h^t(A)$ ={$X^{h(x)(t)}(x)$: x∈A}. For convenience, we set h(x)(t)=h_x(t), for all x∈A and t∈R. We say that a flow X^t is continuum-wise expansive if for any $\epsilon > 0$ there is a $\delta > 0$ such that if $A \subset M$ is a continuum and h∈H(A) satisfies diam$\Upsilon_h^t(A) < \delta$ for all t∈R then A⊂ $X^{(-\epsilon, \epsilon)}(x)$, for some $x \in A$.
A closed invariant set $A \subset M$, A is R-robust continuum-expansive if there are C^1 neighborhood $U(X)$ of X and a residual set R of $U(X)$ such that for any $Y \in U(X)$, $A(Y)$ is continuum-wise expansive. Then we show the followings.

Theorem 1. If the chain recurrence set R(X) is R-robustly continuum-wise then R(X) is hyperbolic.

Theorem 2. If the homoclinic class H(p) is H(p) is R-robustly continuum-wise then H(p) is hyperbolic.

3. Conclusions – In the paper, we proved that if the chain recurrence set R(X) is a type of robust property then it is hyperbolic. We know that if R(X) is hyperbolic then it is Axiom A without cycles. Moreover, if the homoclinic class H(p) is a type of robust property then it is hyperbolic. In the flows, many results are related to singular points. In the paper, we can deleted singular points. So, we consider hyperbolic structure under the assumption.

5. References
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