A New Numerical Scheme to Solve the Hyperbolic Telegraph Equations

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1. Introduction – In this research, a new method is proposed to solve the higher dimension hyperbolic telegraph equations with Dirichlet boundary conditions. The truncated Hermite series with unknown coefficients are used to approximate the solution in both of the spatial and temporal variables. The basic idea for discretizing the higher dimension telegraph equations is based on the collocation method together with the Hermite operational matrices of derivatives. The resulted systems of linear algebraic equations are solved by Tri-Diagonal Matrix Algorithm TDMA. The proposed method is proved to be accurate and it converges fast when it is compared to some of the existing numerical methods ([1], [2], [3]).

2. Experimental – One of the several numerical examples that was solved in this paper is:

$$\frac{\partial^2 w(x,y)}{\partial^2 y} + \frac{\partial \ \ w(x,y)}{\partial \ \ y} + w(x,y) = \frac{\partial^2 w(x,y)}{\partial^2 x}$$

Subject to some initial and boundary conditions.
The numerical approximation is very accurate compared to [3] and its error is around $10^{-9}$for $n_x = n_y = 16$.

3. Results and Discussion - when increasing the number of collocation points the error decreases exponentially which shows that the method has the spectral accuracy for the hyperbolic telegraph equations.

4. Conclusions – The main conclusion of this research is that the time required for the solution to converge is less than that reported by other existing numerical methods. Moreover, a small number of collocation points is required for the solution to converge.

5. References

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